Sieving for Shortest Vectors in Ideal Lattices (work in progress)

Michael Schneider, TU Darmstadt

mischnei@cdc.informatik.tu-darmstadt.de

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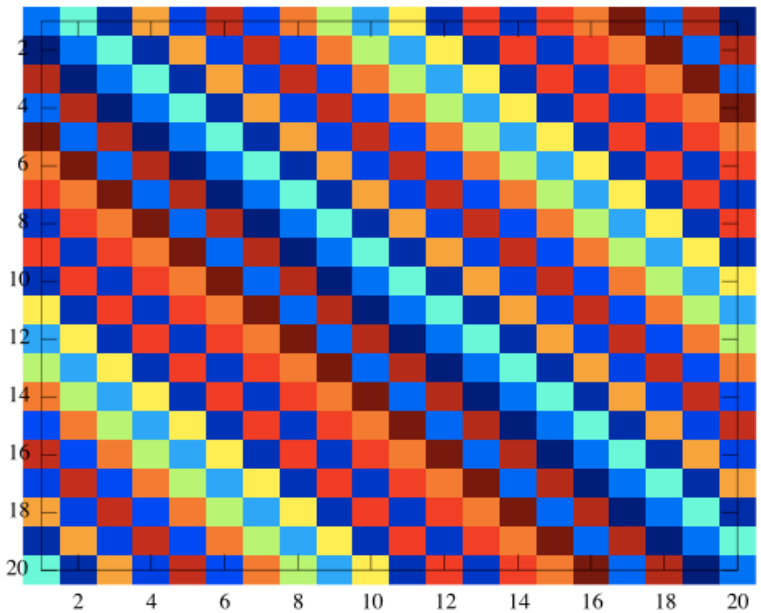

PQCrypto 2010 - Recent Results Section

Shortest Vector Problem

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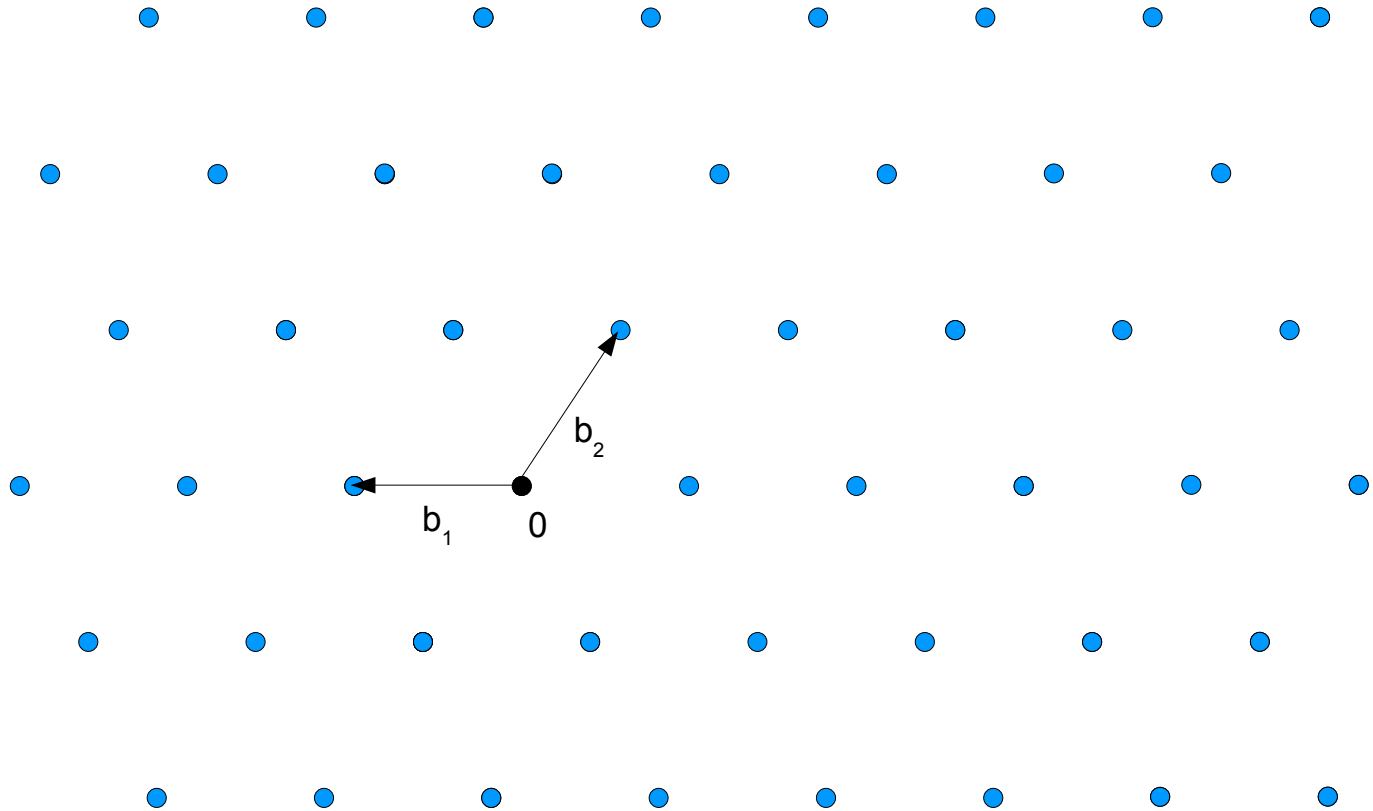

Shortest Vector Problem

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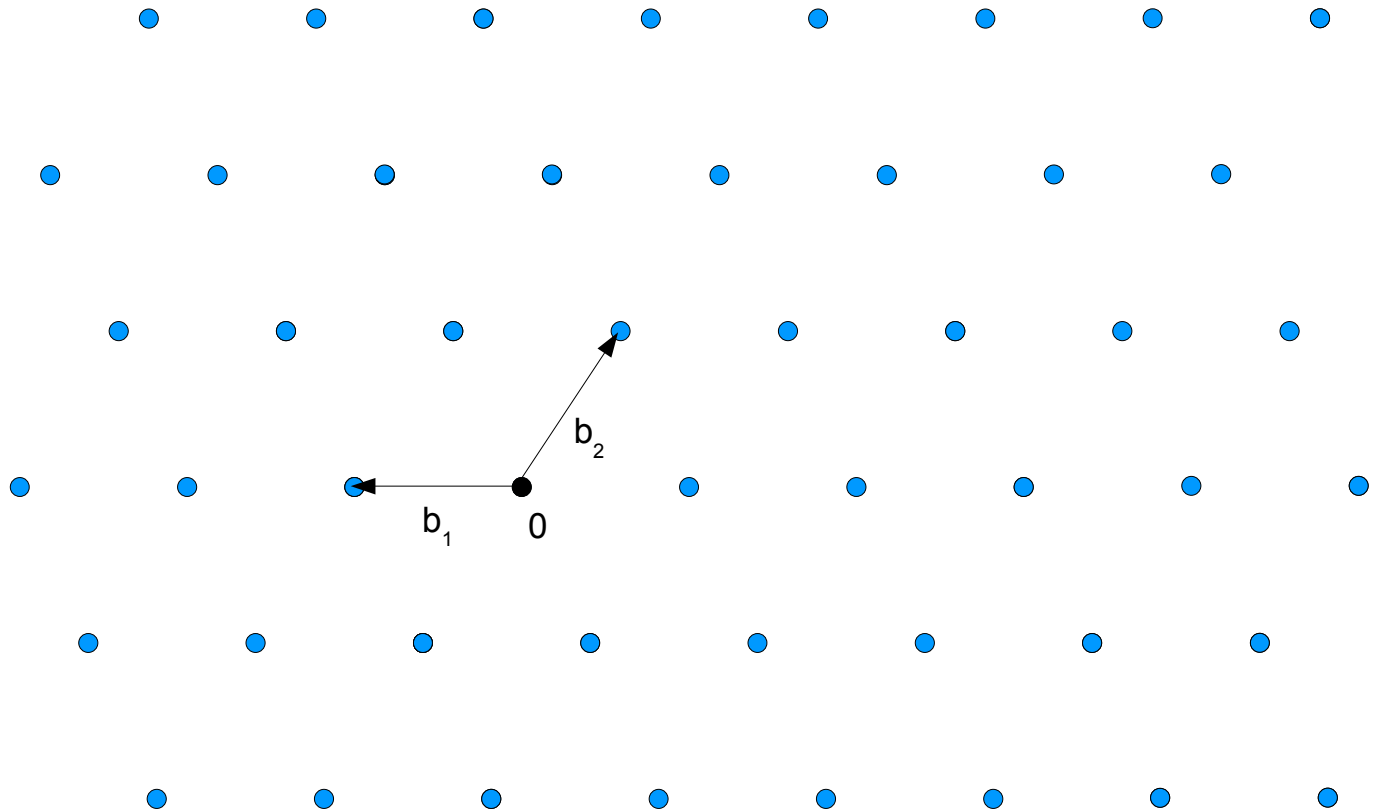

SVP: Given a basis of L, find $\mathbf{v} \in L$ with $\|\mathbf{v}\| = \min_{\mathbf{x} \in L} \|\mathbf{x}\|$

Ideal Lattices

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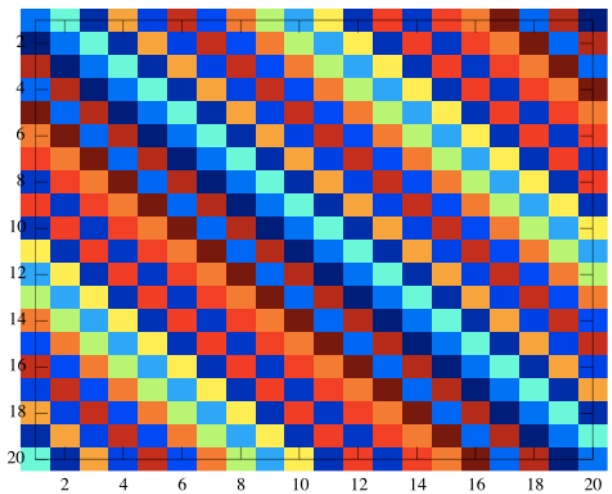
- Lattices of special form
- Used preferably for lattice crypto
- Supposed to be as hard as regular lattices
- For $\mathbf{v} \in L$, also $rot^i(\mathbf{v}) \in L \, \forall \, i = 1 \ldots n-1$



Sieving Algorithms

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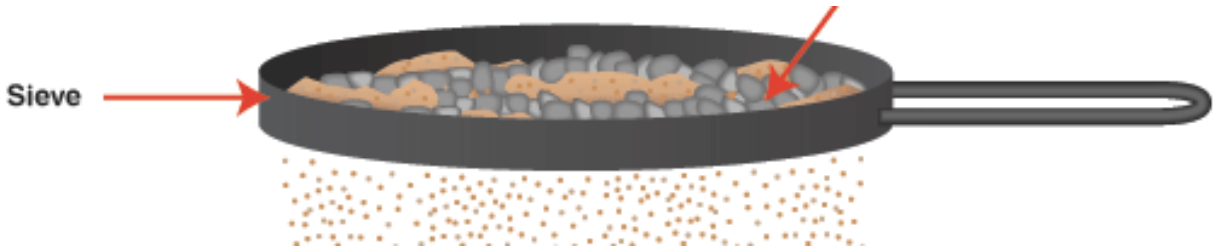
- SVP solver
- Probabilistic
- Exponential runtime $2^{\mathcal{O}(n)}$
- Exponential space $2^{\mathcal{O}(n)}$



Ideal Sieving

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Use

Ideal Sieving

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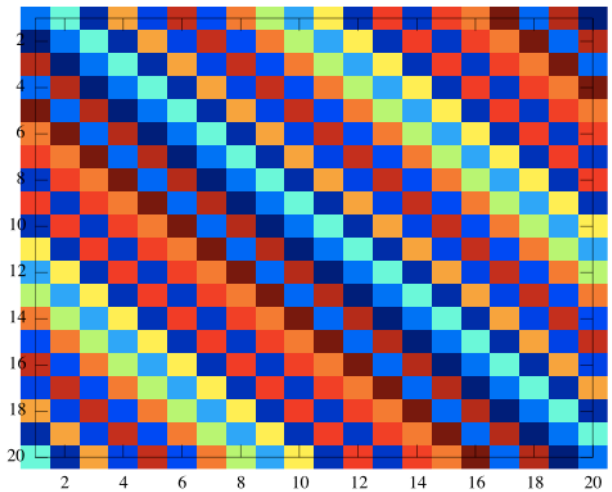

Ideal Sieving

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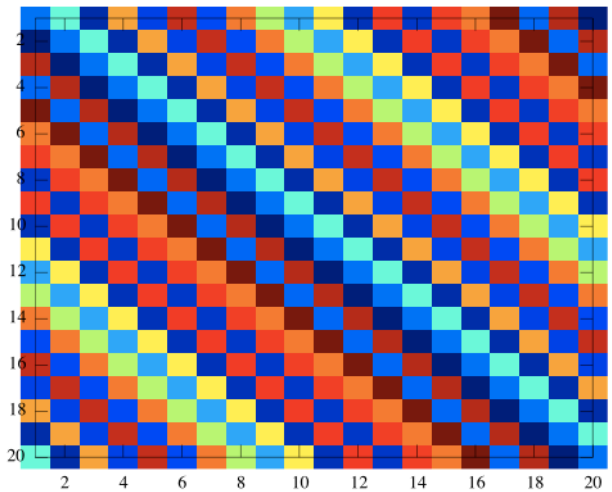
Use



to speed up

Ideal Sieving

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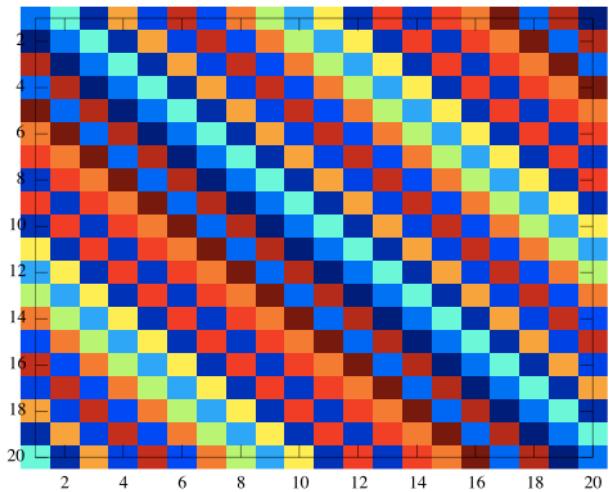
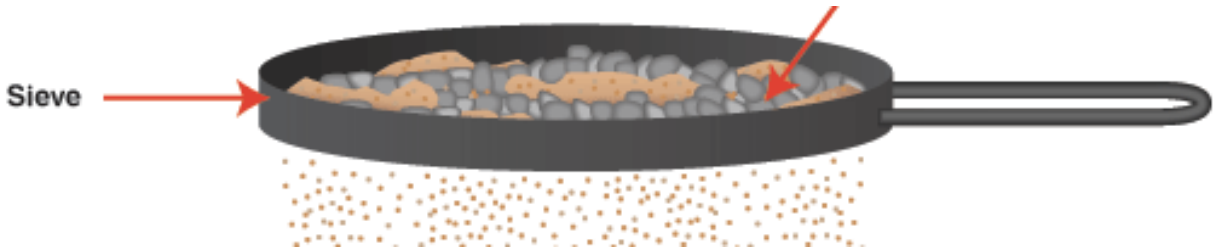
Use



to speed up



Sieving Algorithms

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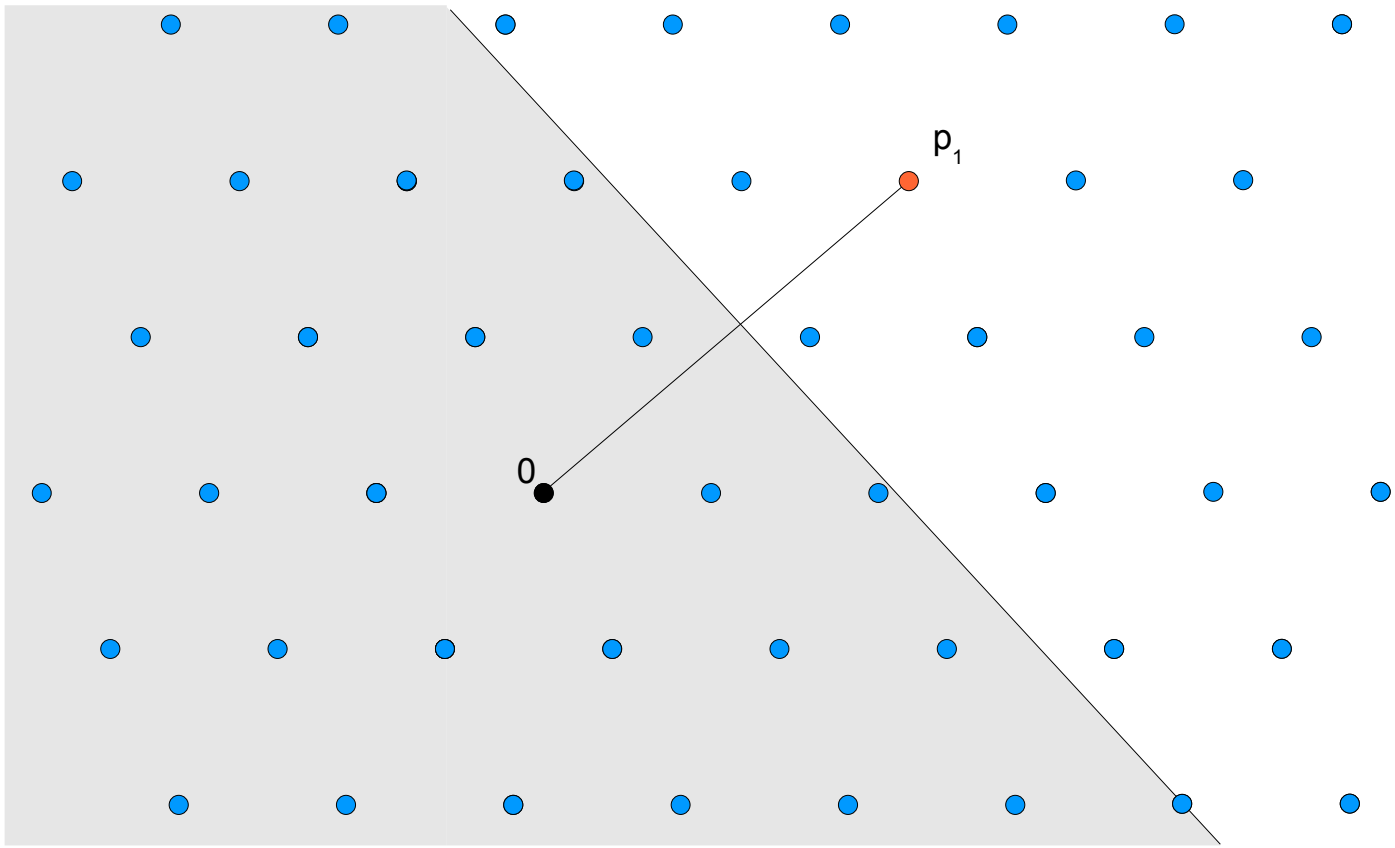

$$\cdot = \{\mathbf{x} : \|\mathbf{x} - \mathbf{p_1}\| \geq \|\mathbf{x}\|\}$$

Sieving Algorithms

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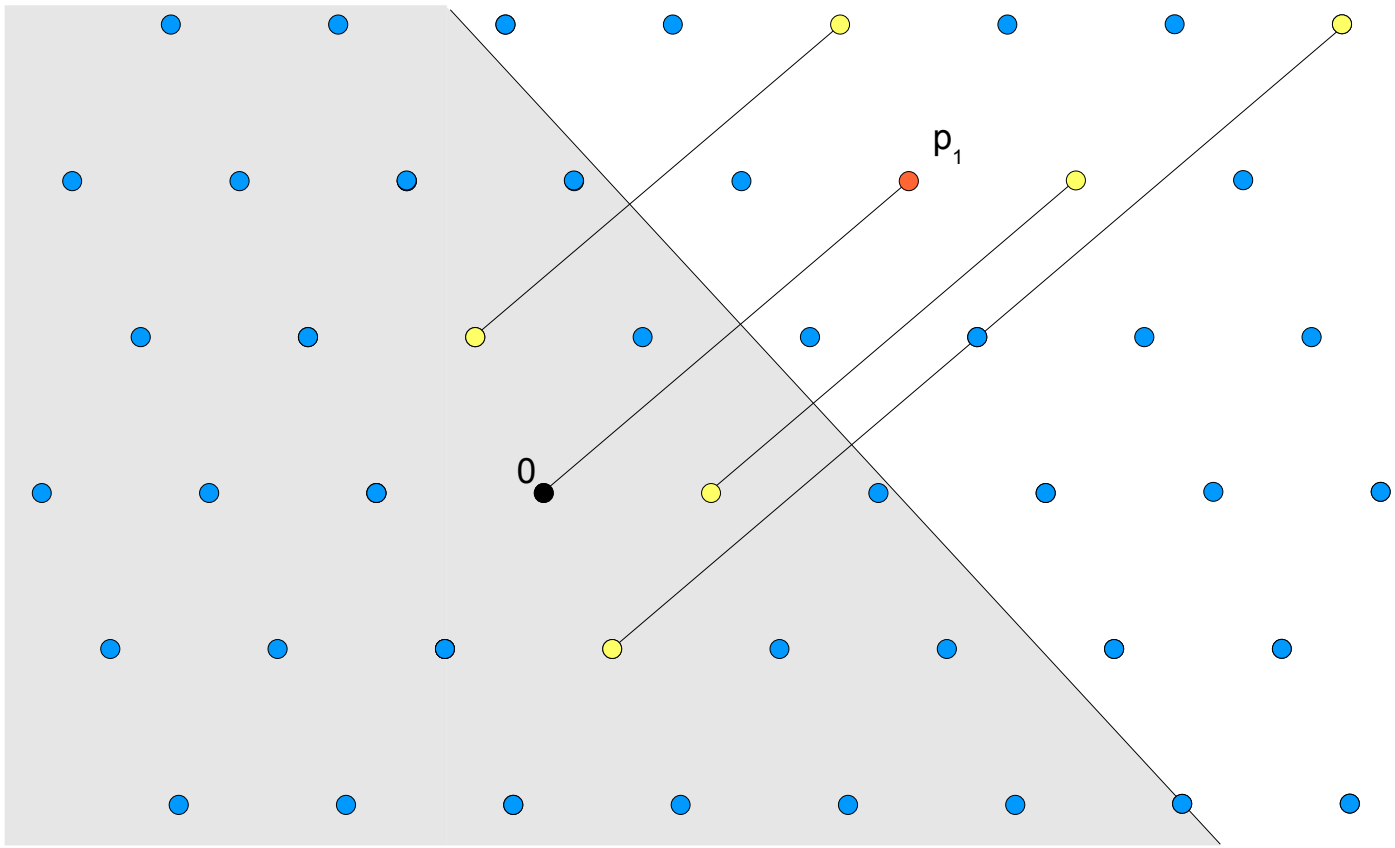

reduce points

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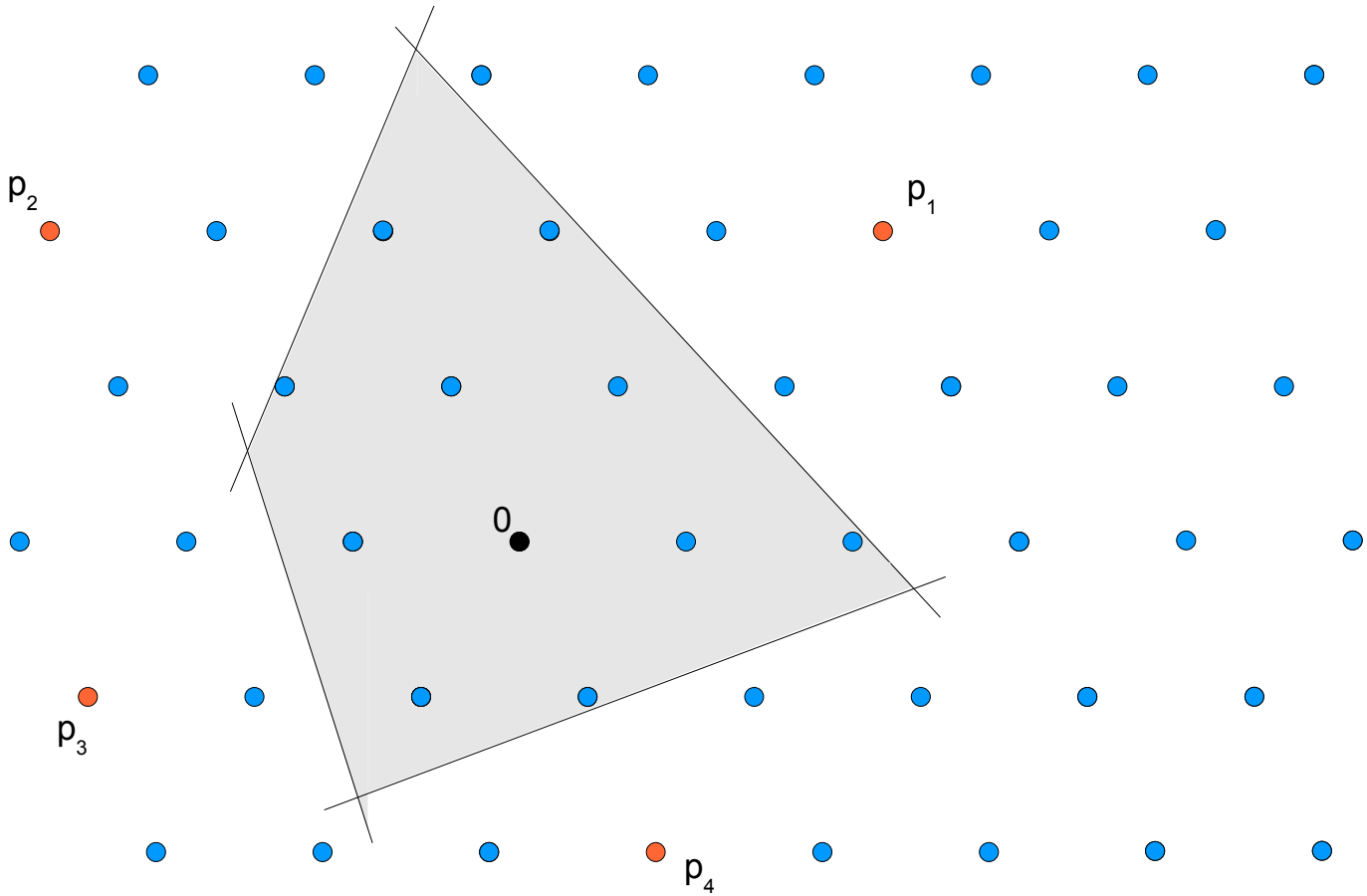

intersection of halfspaces

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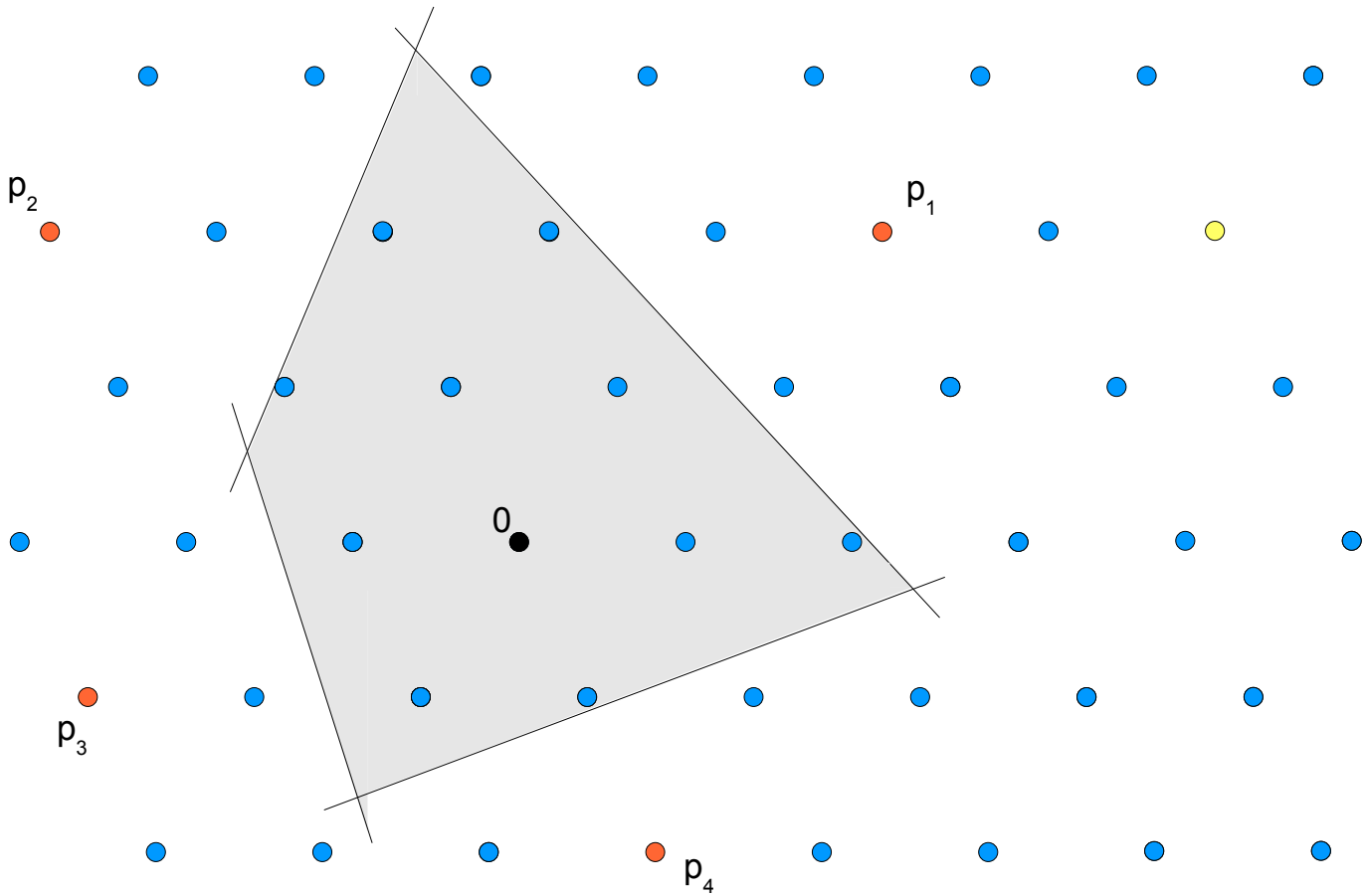

Sieving Algorithms

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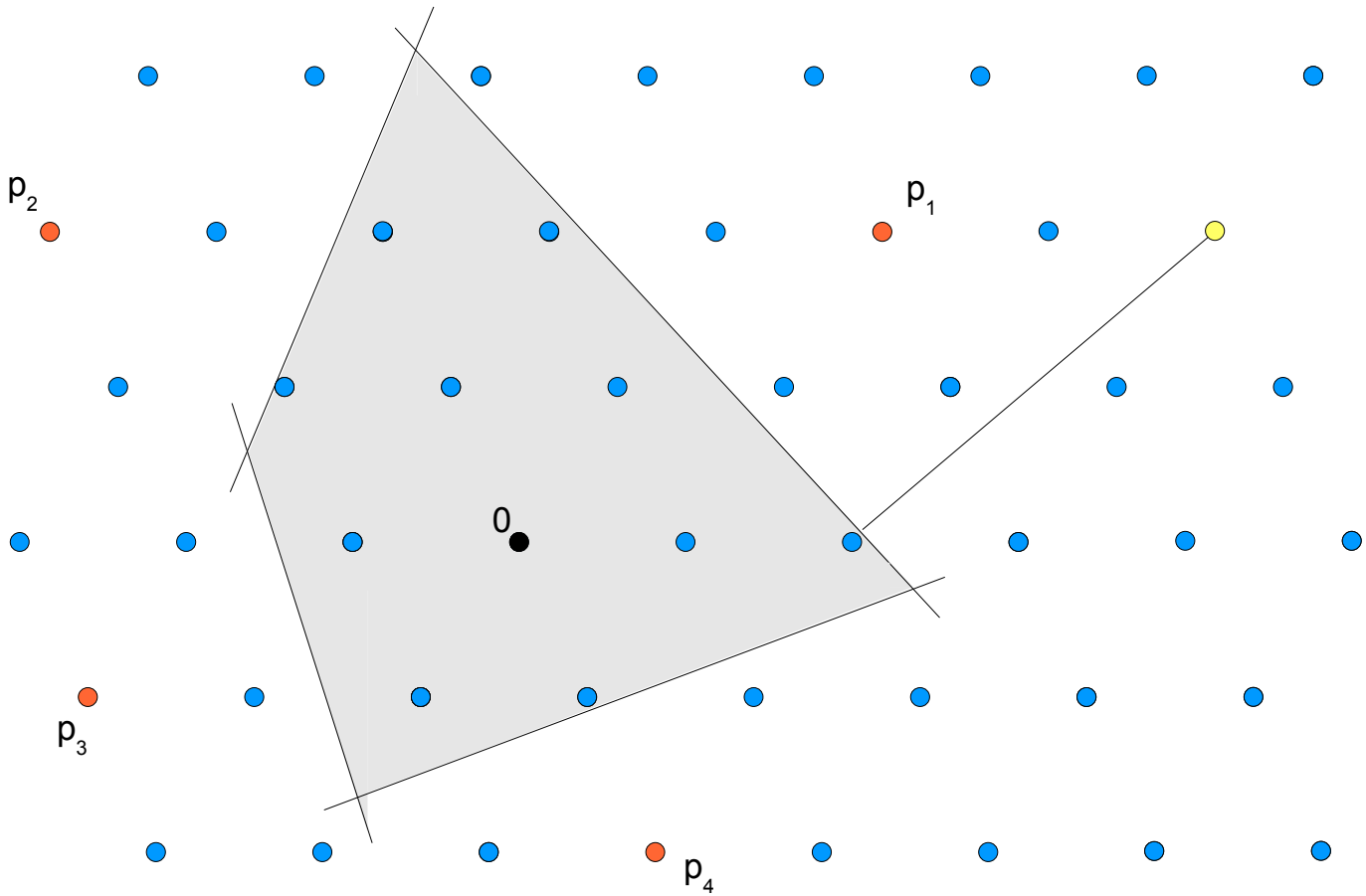

Sieving Algorithms

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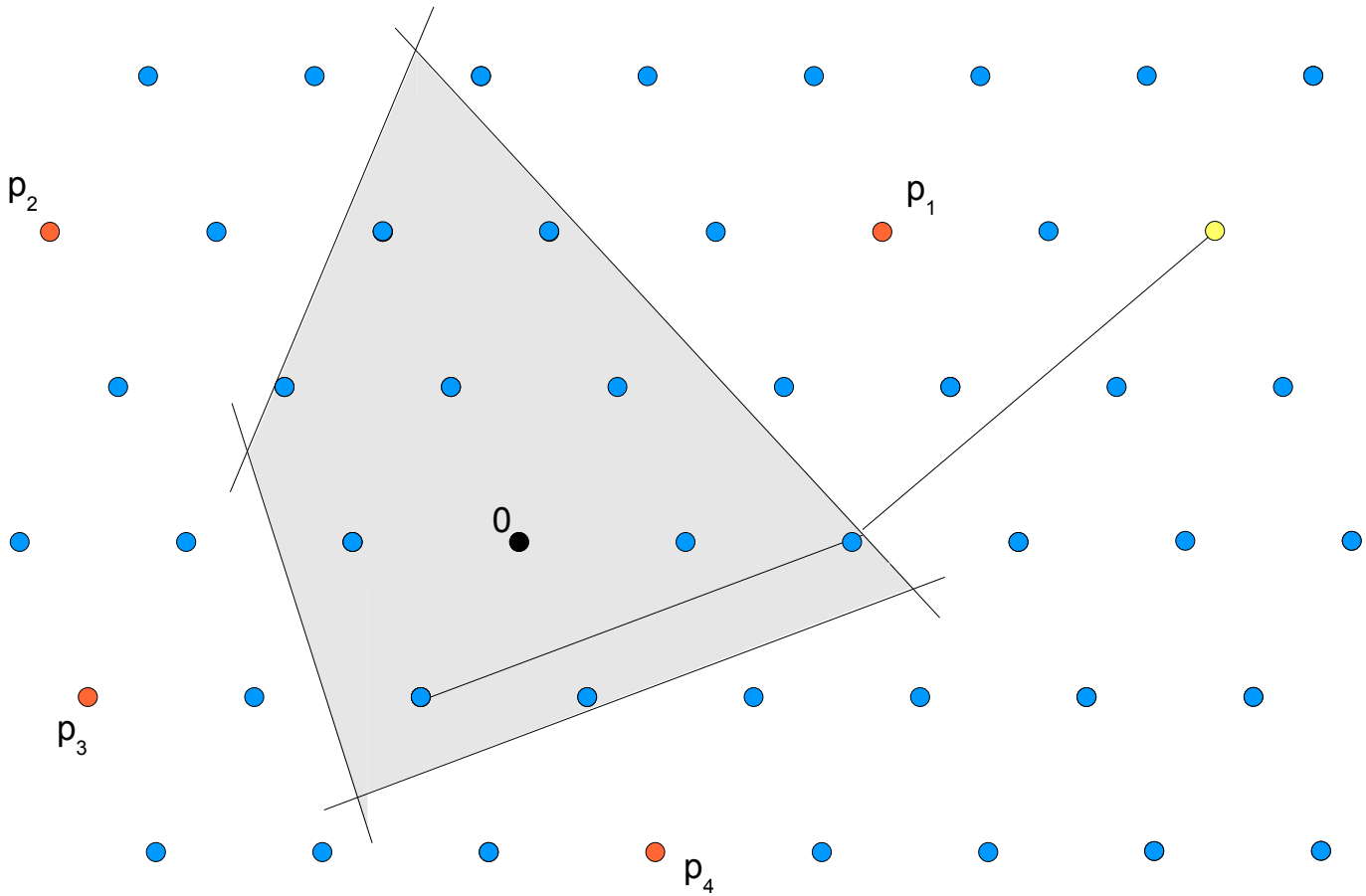

Sieving Algorithms

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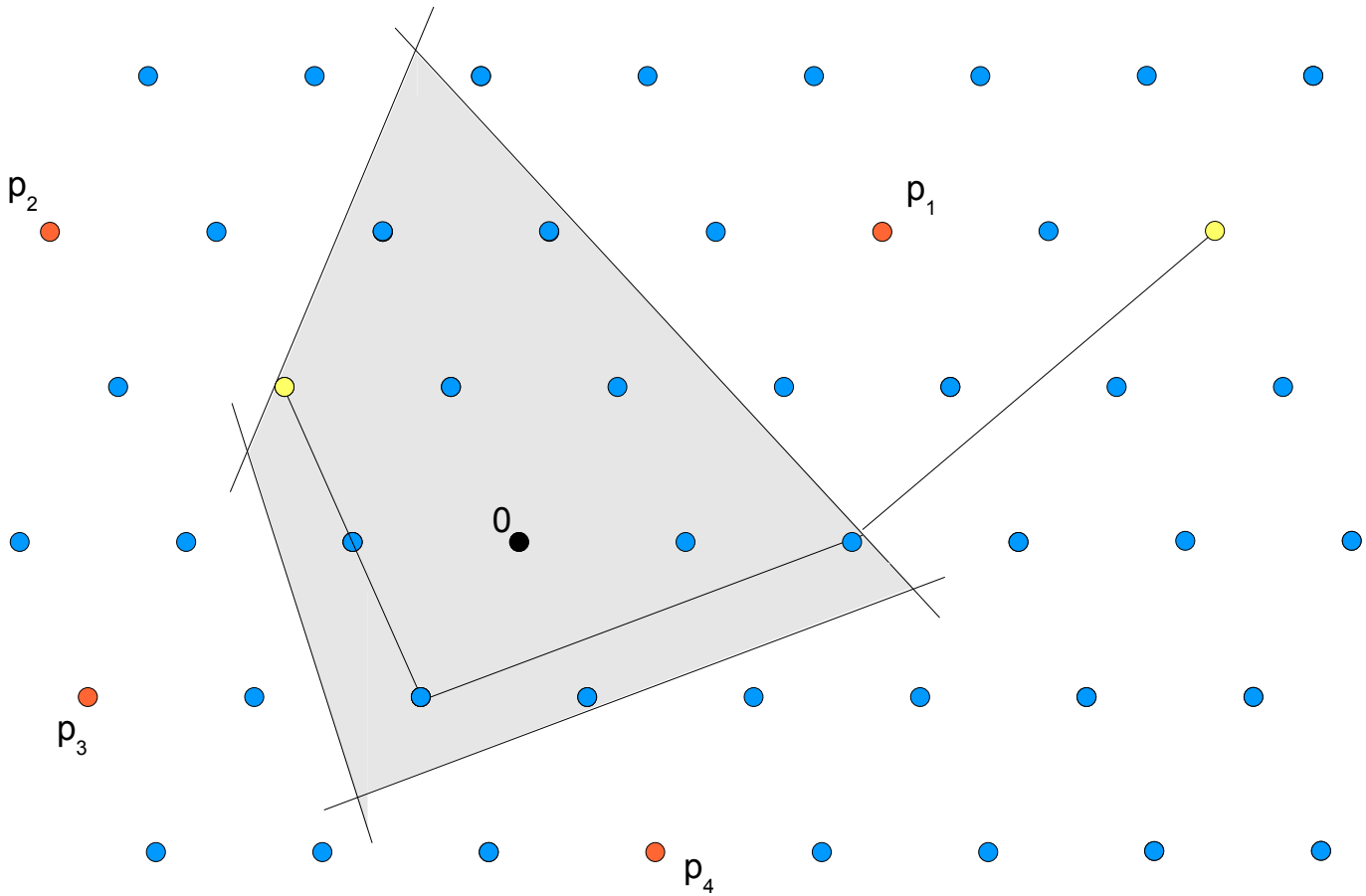

Idea of Ideal Sieving

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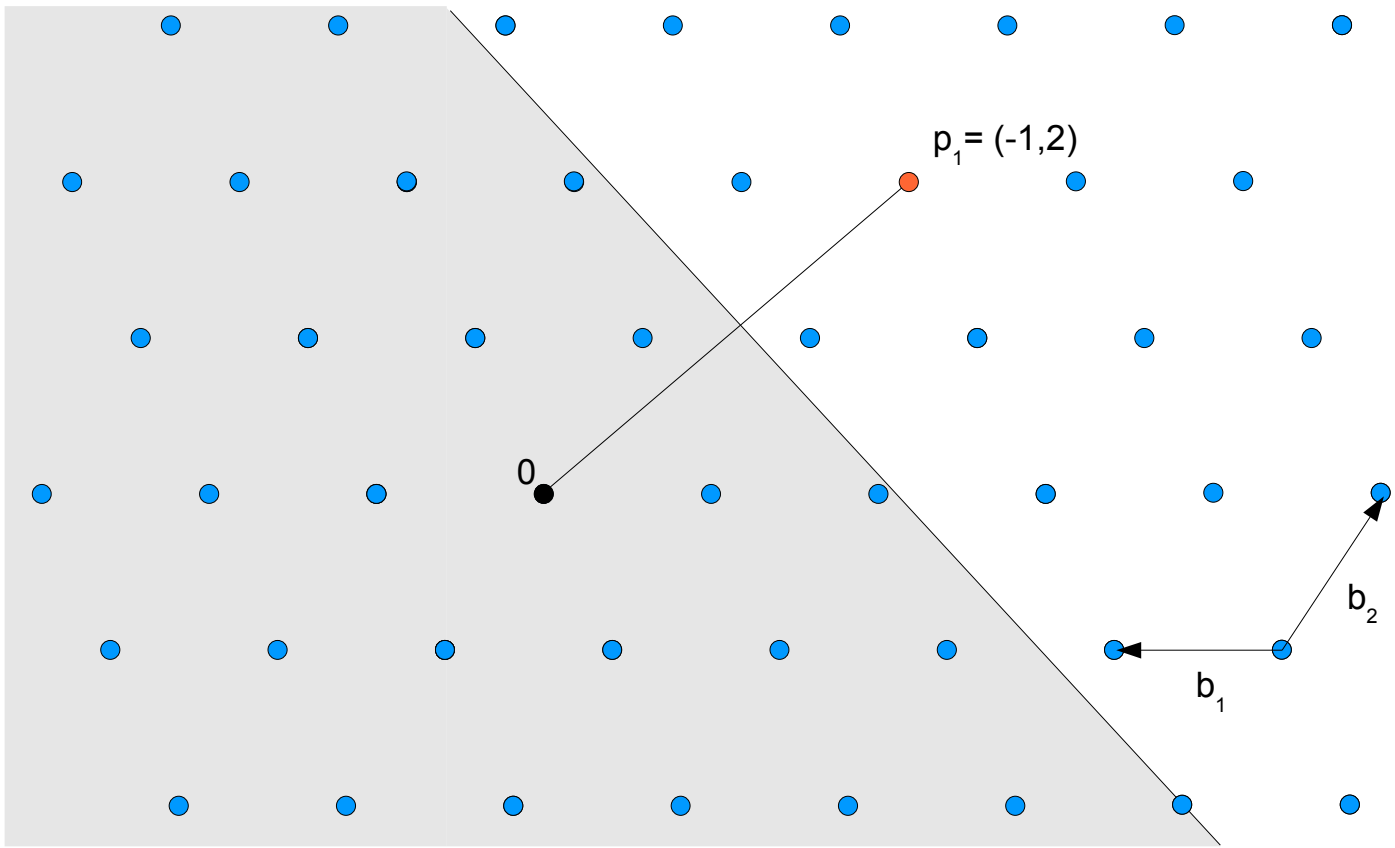

Idea of Ideal Sieving

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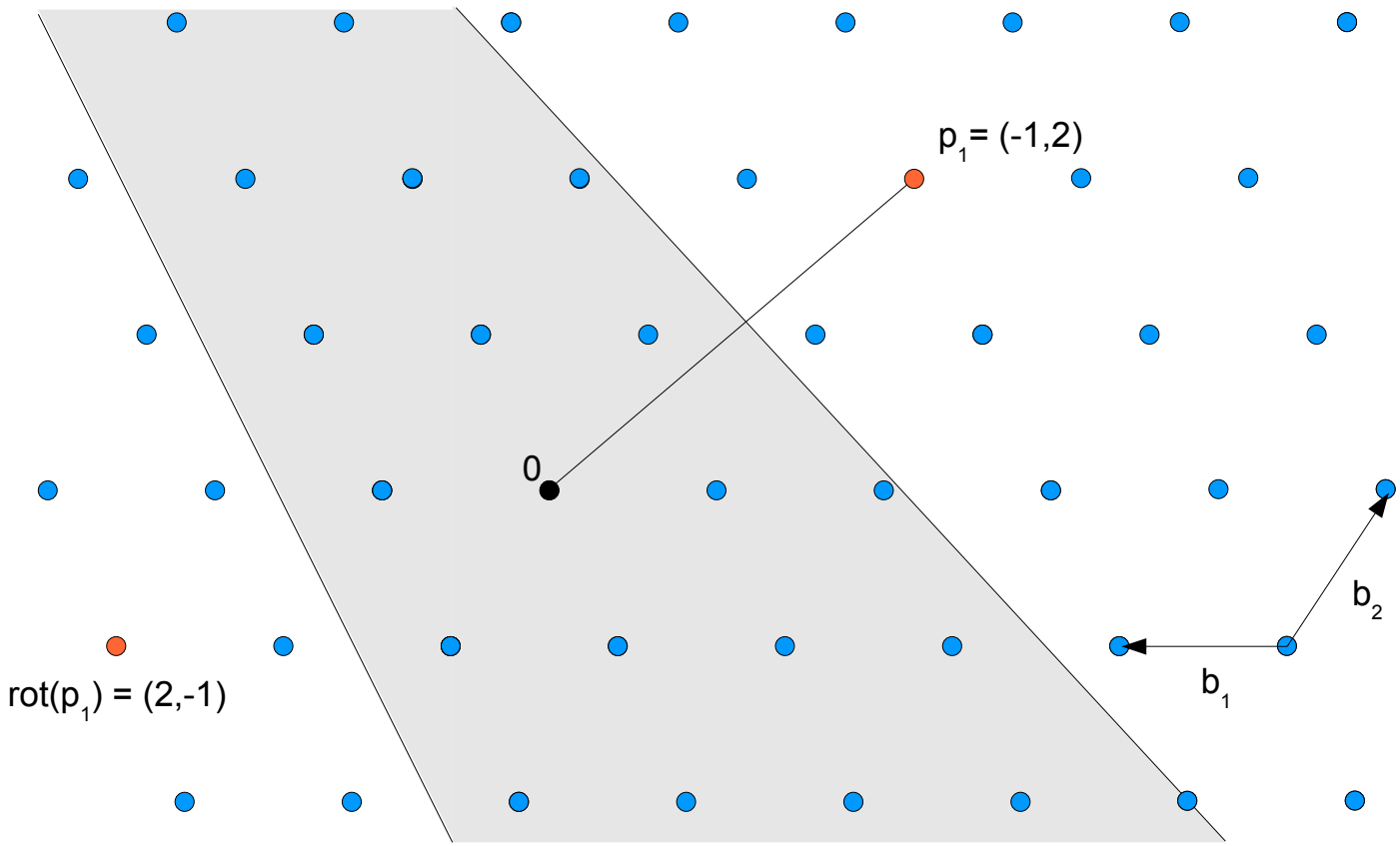

Sieving Algorithms

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Algorithm	Time	Space	Comment
[AKS01]	$2^{\mathcal{O}(m)}$	$2^{\mathcal{O}(m)}$	
[NV08]	$2^{5.9m}$	$2^{2.95m}$	practical
[MV10]	$2^{3.2m}$	$2^{1.33m}$	
[PS10]	$2^{2.46m}$	$2^{1.23m}$	best in theory
[MV10]	$2^{0.52m}$	$2^{0.2m}$	not provable
this work	$2^{0.41m}$	$2^{0.16m}$	only ideal lattices

Experimental Results

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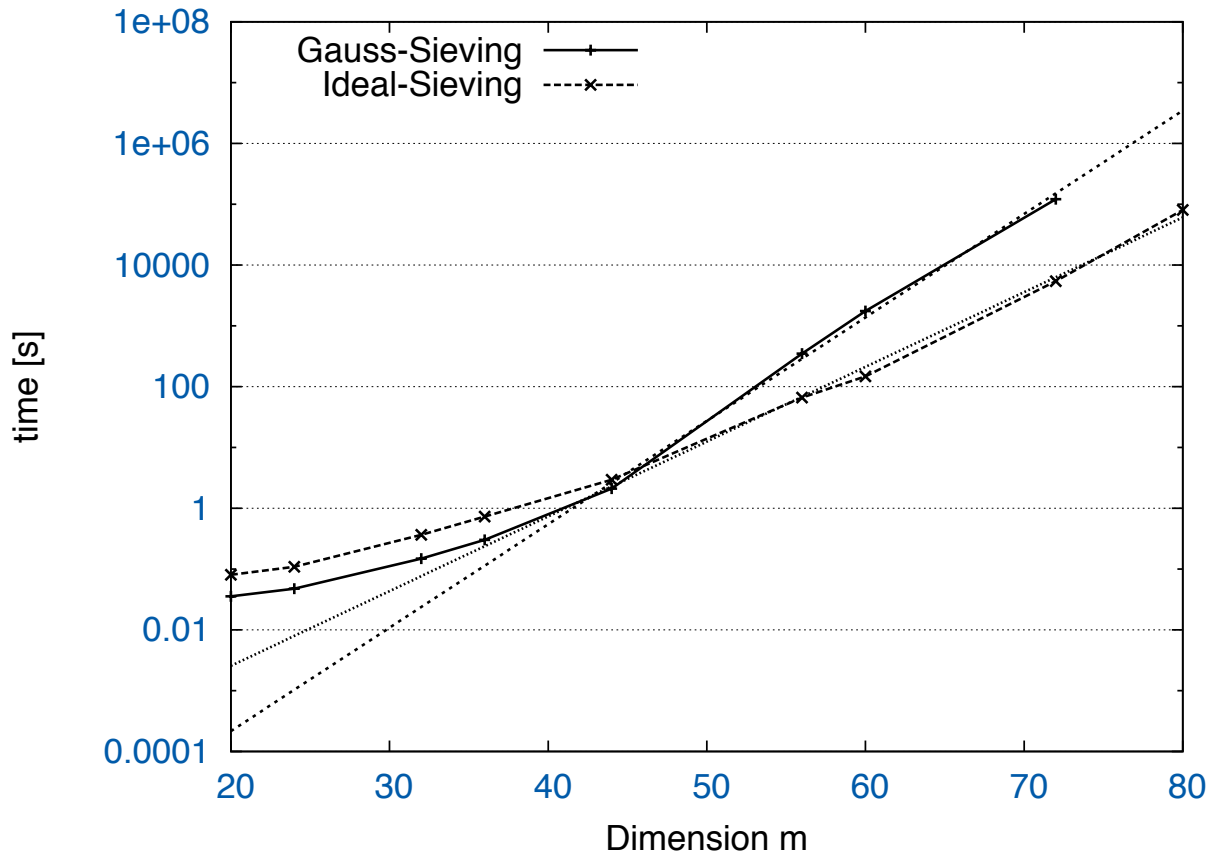
Measured Runtime



Experimental Results

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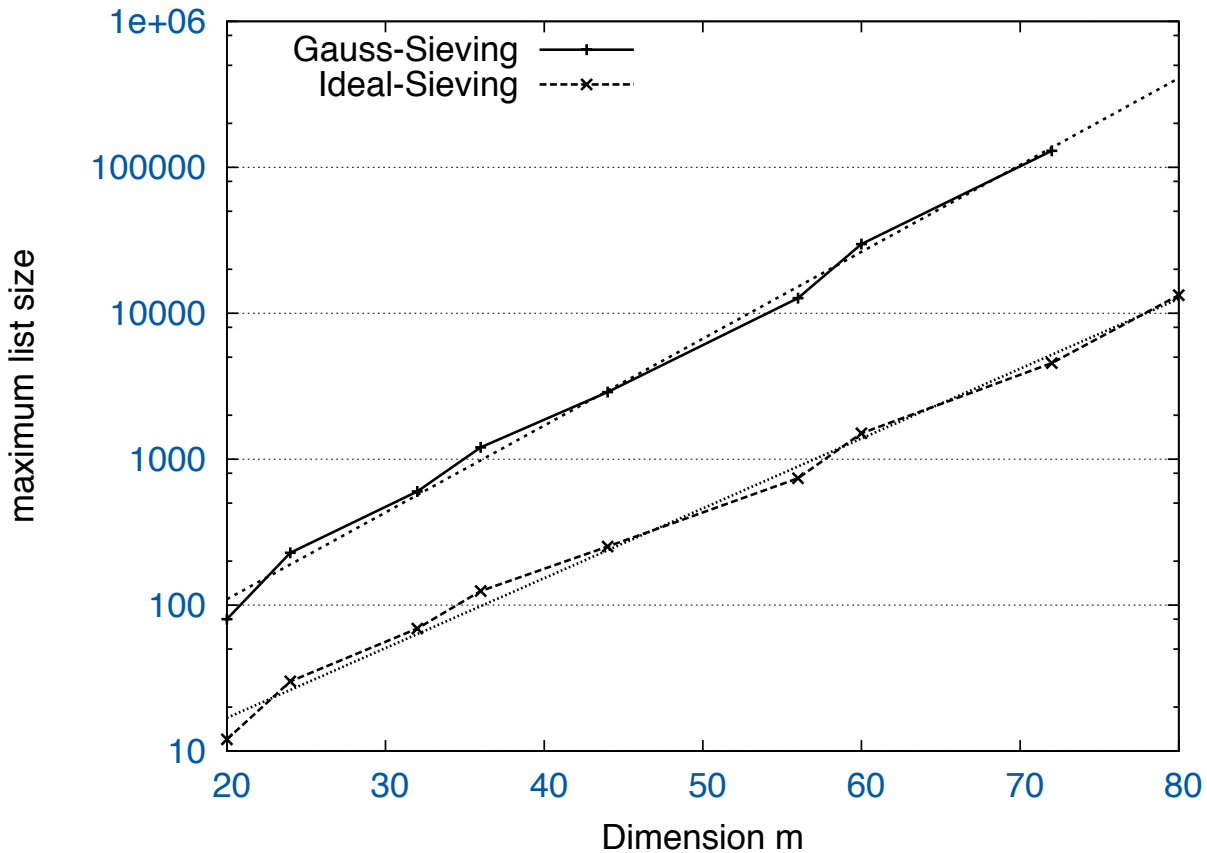
Maximum List Size



Experimental Results

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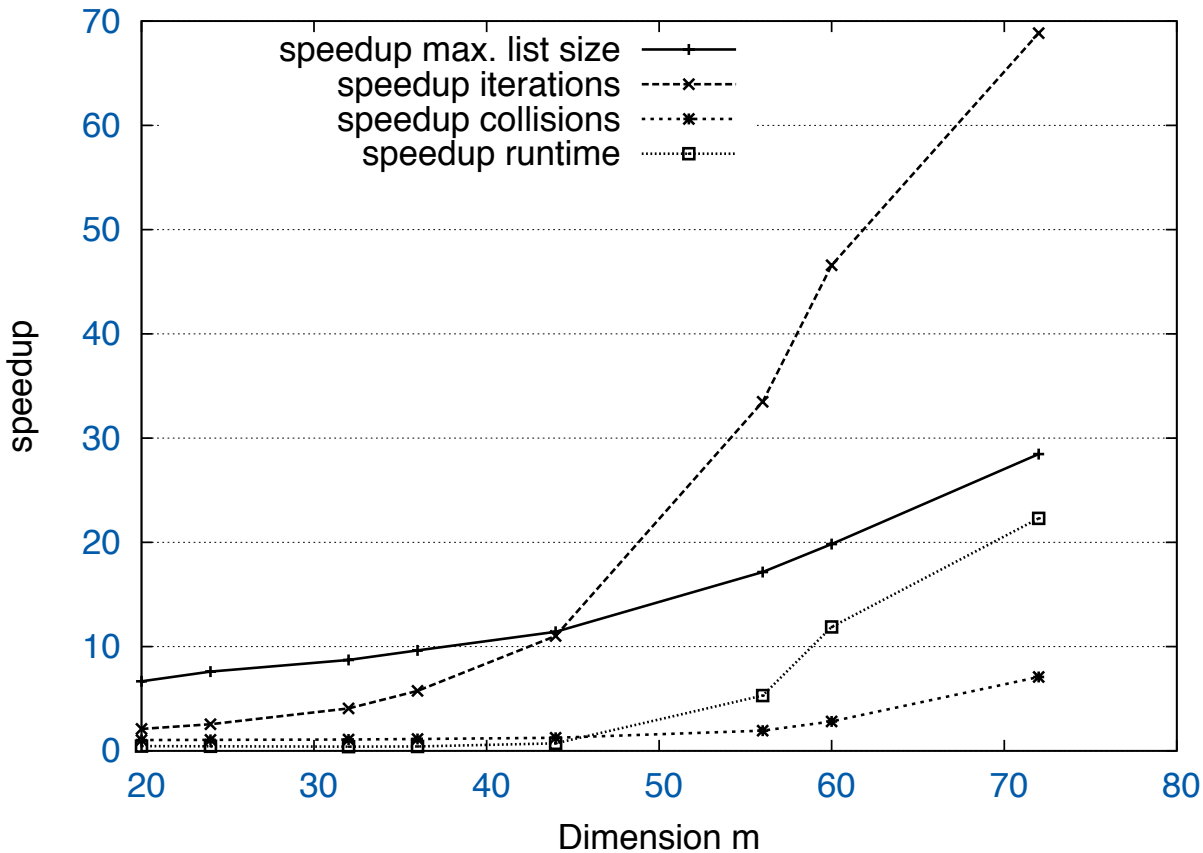
Speedups



Future Work

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- Adapt runtime proof
- ...

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- Adapt runtime proof
- ...

Thank you for your attention!

Bibliography I

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Miklos Ajtai, Ravi Kumar, and D. Sivakumar.

A sieve algorithm for the shortest lattice vector problem.
In *STOC 2001*, pages 601–610. ACM, 2001.

Daniele Micciancio and Panagiotis Voulgaris.

Faster exponential time algorithms for the shortest vector problem.
In *SODA 2010*, pages 1468–1480, 2010.

Phong Q. Nguyen and Thomas Vidick.

Sieve algorithms for the shortest vector problem are practical.
J. of Mathematical Cryptology, 2(2), 2008.

Xavier Pujol and Damien Stehlé.

Solving the shortest lattice vector problem in time $2^{2.465n}$, 2010.
submitted.